Differentials

- A differential is a device that allows a difference in velocity between two rotating elements. Planetary gear trains are typically used to create these mechanisms.
- The most common application of differentials can be found on automobiles.

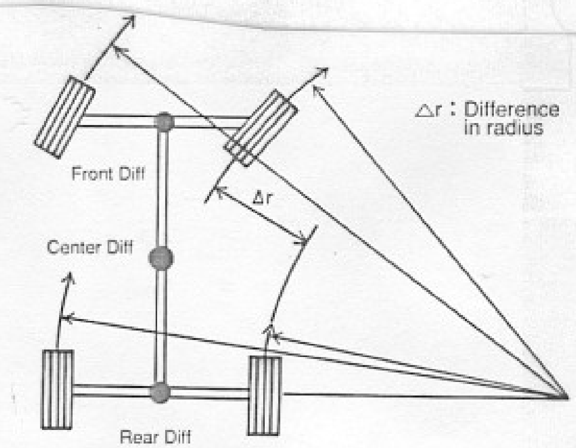
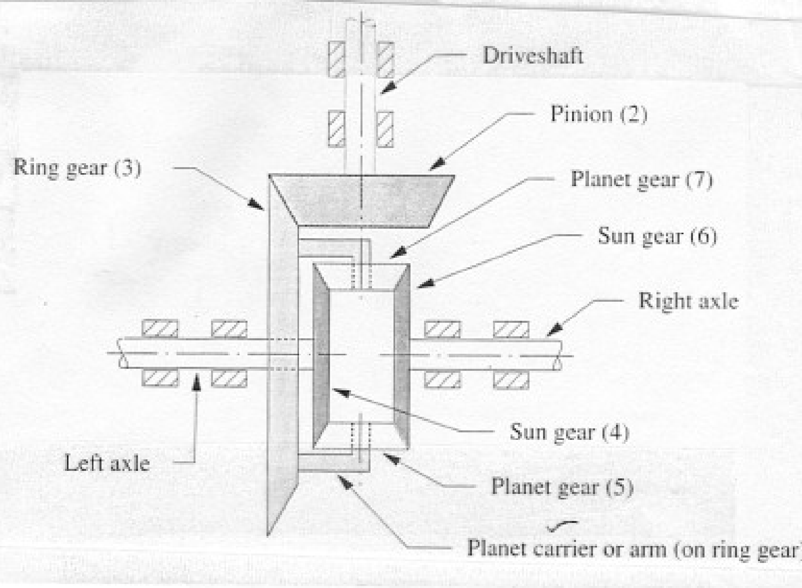


FIGURE 9-50
Turning behavior of a four-wheel vehicle *Courtesy of Tochigi Fuji Sangyo, Japan*

- When a four wheeled vehicle turns, the wheels on the outside of the turn have to travel further, and so they do not rotate at the same speed as the inside wheels. If the wheels were coupled together using a solid shaft, the wheels would either slip or would make it hard to turn.

FIGURE P9-3
Automotive differential planetary gear train



- For all wheel drive (AWD) vehicles there may also be a difference in velocity between the front and rear wheels. In order to account for this an additional center differential is needed.

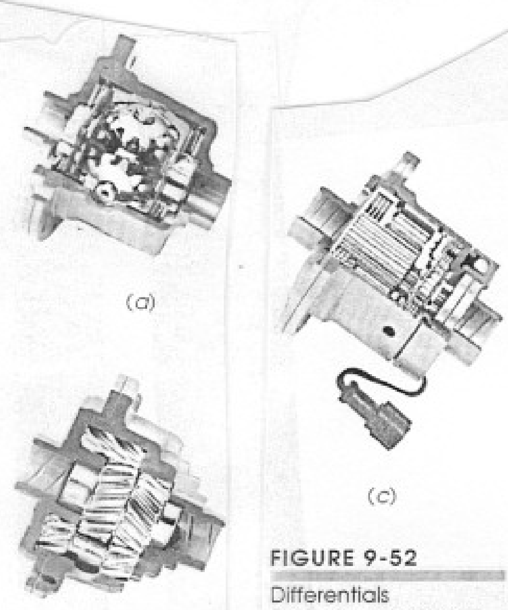


FIGURE 9-52
Differentials
Courtesy of Tochigi Fuji Sangyo, Japan

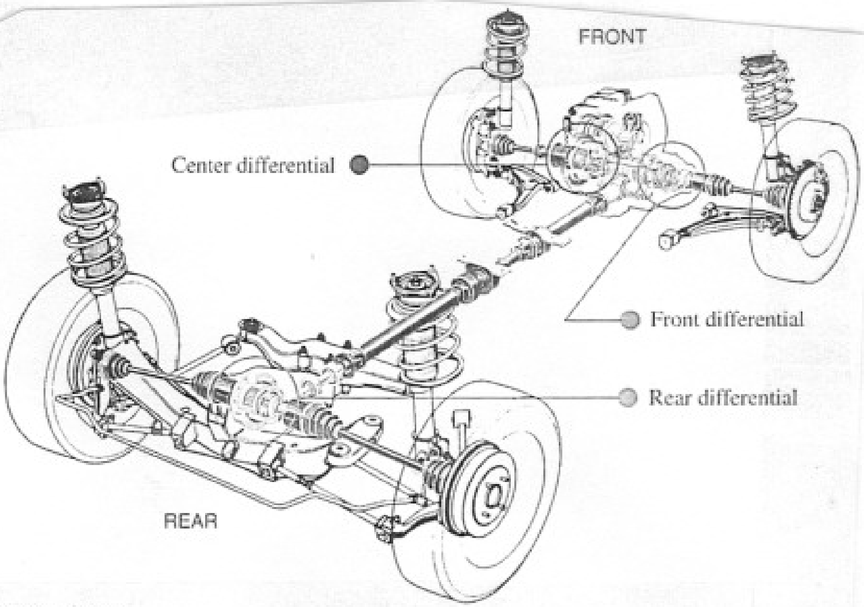


FIGURE 9-51
An all-wheel-drive (AWD) chassis and drive train *Courtesy of Tochigi Fuji Sangyo, Japan*

- Older trucks typically operated with only two wheel drive. When the road conditions became slipery, the hubs on the front axle needed to be engaged to have the vehicle operate in 4WD.

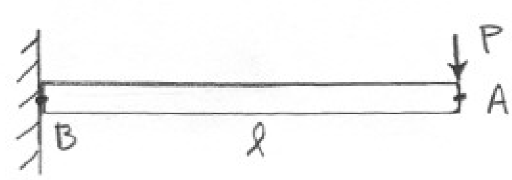
- Differentials split the torque equally between their two wheel outputs. Power = (Torque)(angular velocity)
Since the power is fixed by the engine, if one wheel loses traction (as on ice) it gets all the power (50% torque x 200% speed) and the other wheel with traction gets zero power (50% torque x 0% speed). One wheel spins and the other does not rotate

- For 4WD or AWD vehicles the center differential splits the torque between the front and rear wheels in some proportion.

- Even for 4WD vehicles (with standard differentials) if one front and one rear wheel loses traction, the vehicle will not have control. Several designs called "Limited Slip Differentials" have been created to limit the slip between the two outputs. Typically they have a friction device or a clutch that couples the two output gears to transmit torque while allowing slip for turning.

Dynamic Force Analysis

- Typically we design mechanisms to have a prescribed motion (displacement, velocity or acceleration). Once we know the motion, we can use a Kinetostatic or inverse dynamic solution to determine the forces and torques.
- "Dynamics" solves the Kinematics by knowing the forces
- To solve a Kinetostatic problem, we need to know the following: Kinematics (linear and angular acceleration of all CG's), mass of each member, moment of inertia wrt. each member's CG.
- Let's consider a static case first. The beam is in static equilibrium



To find the reactions at B we need to sum forces and moments

$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M = 0$

- To solve for the forces for a dynamic problem, we can use Newton's Law. For a rigid body

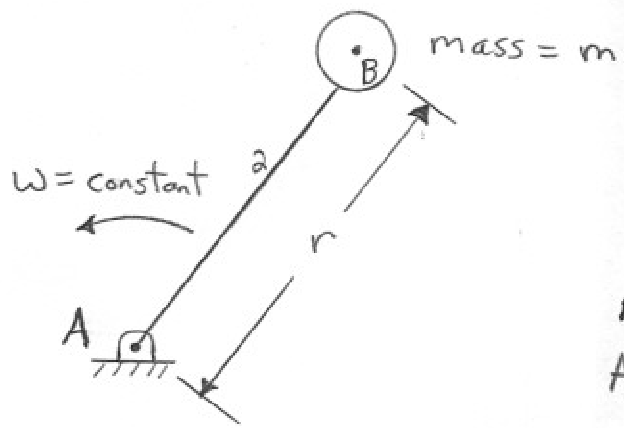
$$\Sigma F_x = m a_x \qquad \Sigma F_y = m a_y \qquad \Sigma T = I_g \alpha$$

a_x = acceleration of the center of mass in the x-direction

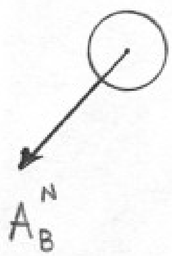
a_y = acceleration of the center of mass in the y-direction

I_g = mass moment of inertia about the center of mass

- Consider a simple pendulum rotating at constant velocity



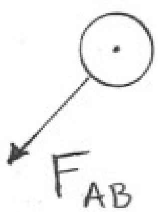
Let's calculate the force in link 2



Because the motion is circular the acceleration is centripetal

$$A_B^N = \omega^2 r$$

To solve for the force we need to draw a FBD (assuming outward is positive) and write Newton's Law



$$\Sigma F_r = -F_{AB} = m a_r = m(-\omega^2 r)$$

$$F_{AB} = m\omega^2 r$$